



# *Pre-Calculus Curriculum Essentials Document*



*Boulder Valley School District  
Department of Curriculum and Instruction  
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### Pre-Calculus Curriculum Essentials

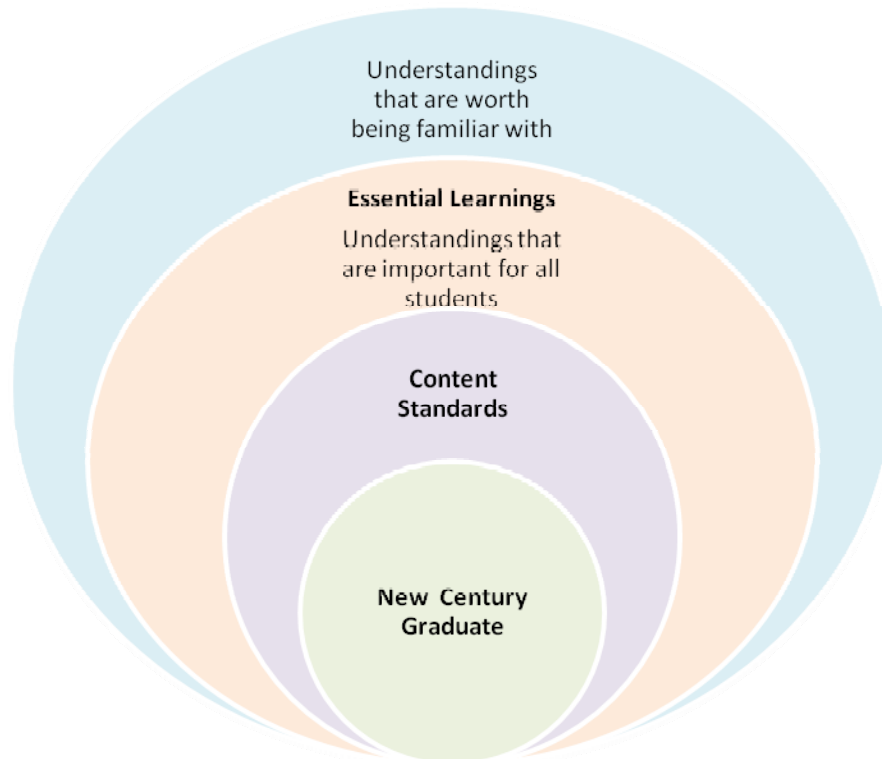
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# *General Introduction*



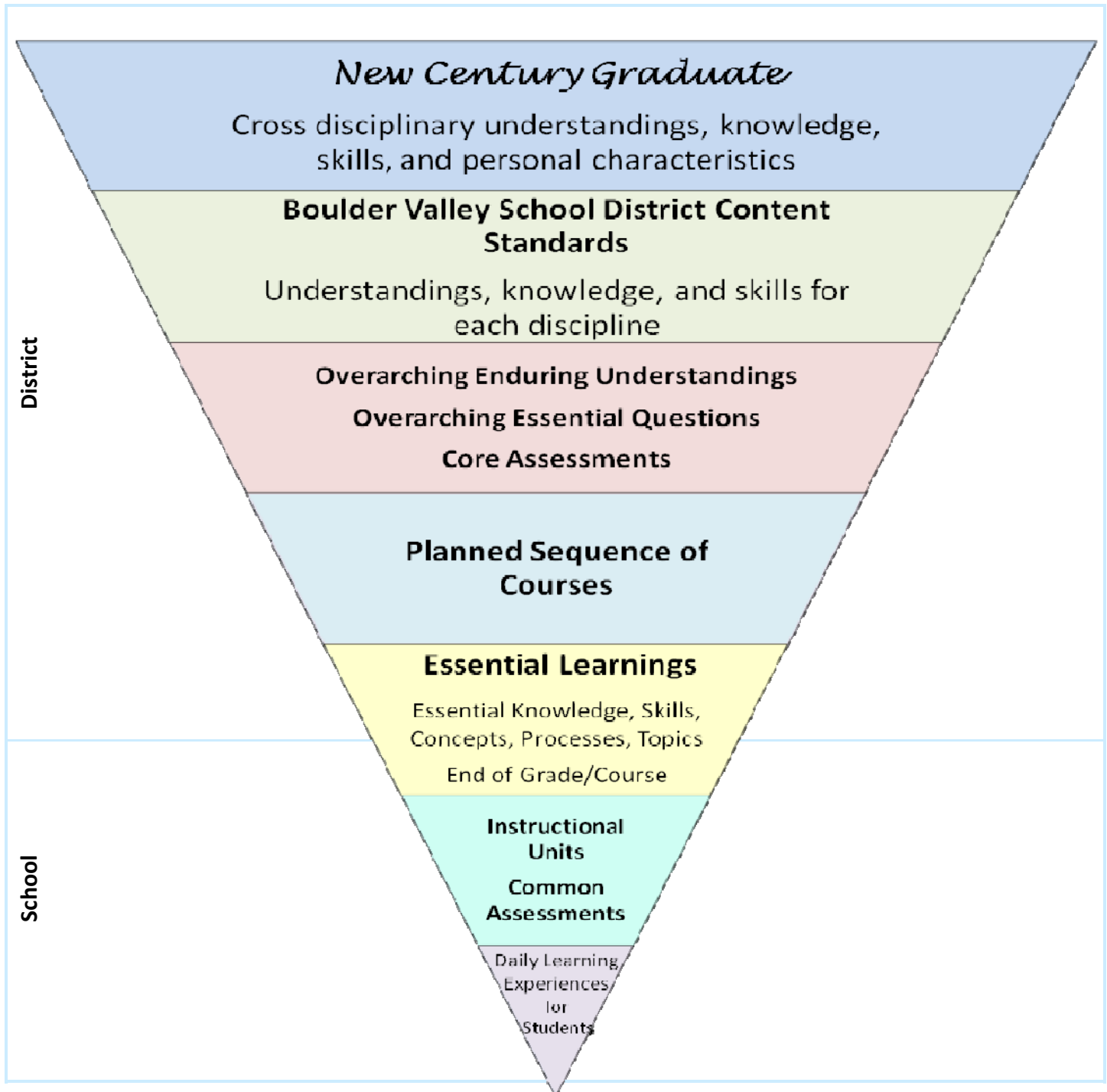
## What is a Curriculum Essentials Document? How Does it Relate to a Guaranteed and Viable Curriculum?



Because we are faced with more content than we can reasonably address, we are obligated to make choices and frame priorities. A useful framework for establishing priorities is graphically depicted using 4 nested ovals. The innermost oval, *New Century Graduate*, represents the goals of schooling that have been identified by the Boulder Valley School District community. Moving to the next oval, *Content Standards*, levels of performance for each program of study are clearly articulated. The third oval, *Essential Learnings*, represents the **viable curriculum**. A curriculum is viable when the number of learnings can be accomplished in the time provided (usually a semester, trimester, or year). Thus, an Essentials Document identifies the priorities for learning that are necessary for successful learning at a particular grade level or course and beyond. It also identifies the essential knowledge, skills, concepts, topics, and processes that support the attainment of the essential learning. Finally, the largest oval represents the field of all possible content that might be examined during a grade level or course. This includes extended learning opportunities for students who have achieved the essential learnings or attending to background knowledge and skills that students may need to review or learn to ensure achievement of grade level or course essential learnings.

## Curriculum Framework: Macro and Micro Levels

The New Century Graduate identifies the knowledge, skills and personal characteristics that our community has identified as the goals of schooling. Programs of study and curricular content are identified and addressed as a means



## *New Century Graduate* Knowledge and Skills

### **Life Competencies**

Leads a balanced life: exhibits physical fitness, knows good nutrition rules, stays safe and drug free, knows how to have fun and relax, manages anger and stress, exhibits self-sufficiency and self confidence, and finishes tasks.

Understands money management, budgeting, balancing a checkbook, debt management, and record keeping.

Demonstrates time management skills and a broad base of knowledge in practical skills such as cooking, sewing, driving, and map reading.

Knows how to search for a job and knows where to go to find answers.

### **Communication: Speaking and Writing**

Writes and speaks thoughtfully and articulately to inform, to express one's thinking and creativity, and to communicate to diverse audiences.

Uses correct grammar, spelling, and mechanics; organizes for effectiveness

Uses technology for effective communication

### **Multicultural/Global Perspective**

Understands global customs, economics, literature, history, politics, religions, geography, and demographics.

Understands the contributions of different cultures to our society

Demonstrates proficiency in a language other than English.

### **Literacy: Reading**

Reads critically, fluently, and with comprehension.

Reads for information research, pleasure and knowledge of literature.

### **Mathematics**

Demonstrates basic math computational skills and understand higher-level mathematical concepts and reasoning.

Understands conservation and resource management.

### **History**

Possesses knowledge of American and World Histories and their influence upon the present and the future.

Employs literature as a tool for learning about history across cultures.

### **Science**

Demonstrates basic sciences knowledge and understands high-level scientific systems including environmental systems.

Knows how to apply the scientific method to real situations.

### **Arts**

Experiences and appreciates music, visual arts, dance and theater.

## *New Century Graduate* Personal Characteristics



### **Respect for Others (Values Others)**

Understands and values differences including: cultural, religious, ethnic, gender, age, and ability.

### **Initiative and Courage**

Exhibits self-motivation, self-discipline, persistence, independence, confidence, curiosity, and willingness to take risks, without being afraid to fail.

### **Citizenship**

Understands his or her role and responsibilities and contributes to the community, nation, and world.

### **Responsibility**

Takes responsibility for own thoughts and actions, accepting the consequences.

### **Ethical Behavior**

Exhibits personal integrity through honesty, fairness, sincerity, and a sense of justice.

### **Flexibility and Open Mindedness**

Demonstrates flexibility, open-mindedness, adaptability, resiliency, and openness to change.

### **Self-respect**

Possesses self-respect and confidence, while recognizing one's own limitations.

## What are Enduring Understandings and Essential Questions?

### **Enduring Understandings**

are the big ideas central to a content area that have lasting value beyond the classroom and are transferable to new situations. Enduring understandings describe what, specifically, students should understand about the topic. Such understandings are generally abstract in nature and are often not obvious, thus requiring uncovering of a topic through sustained inquiry.

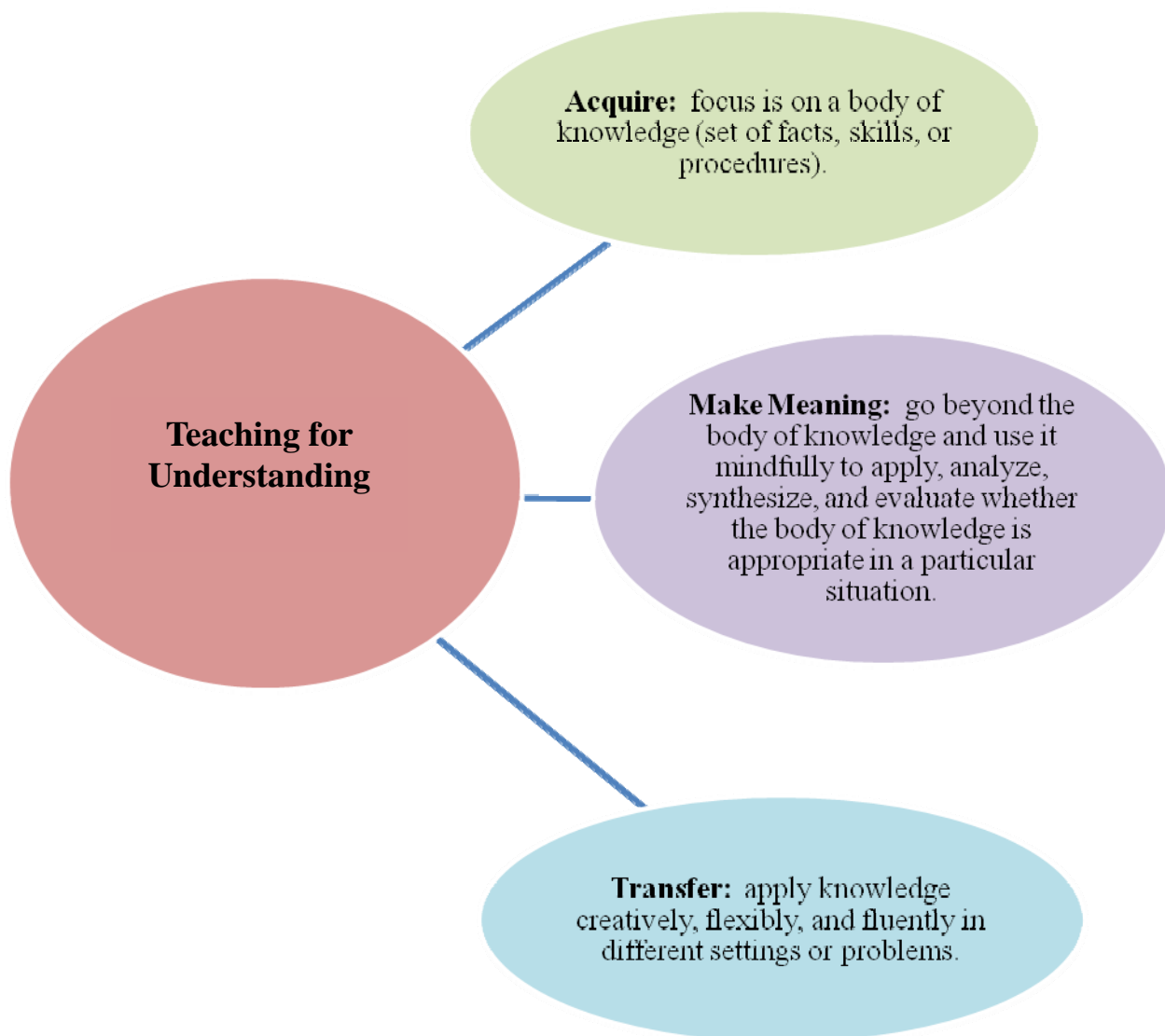
An understanding can be overarching or topical. Overarching understandings are broad (as the name implies) and offer a possible bridge to other units and courses. Overarching understandings are identified at the district-level. Topical understandings are unit specific, identified by teachers about the understandings the unit will cultivate about specific topics.

**Essential Questions** provoke deep thought, lively discussion, sustained inquiry, and new understandings culminating in meaningful performances. They require students to consider alternatives, weigh evidence, support their ideas, and justify answers. Essential questions do not yield a single straightforward answer, but produce different plausible responses, about which thoughtful and knowledgeable people may disagree. Essential questions spark meaningful connections with prior learnings and personal experiences and create opportunities for transfer to other situations and subjects.

An essential question can be either overarching or topical in scope. Overarching essential questions are general in nature, causing genuine and relevant inquiry into the big ideas and core content. They cut across units and/or courses. Topical essential questions focus on a specific topic and meant to be answered—if only provisionally—by unit's end.

## Teaching for Understanding

If learning is to endure in a flexible, adaptable way for future use, then teachers must design units that provide opportunity for students to 1) acquire knowledge; 2) to deepen the meaning of that knowledge by using it mindfully, and 3) to transfer their learning to new situations or problems.



## What Does it Mean to Understand?

### Knowledge

- observation and recall of information
- knowledge of dates, events, places, major ideas
- *Question Cues:* list, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where

### Comprehension

- grasp meaning and predict consequences
- order, group, classify, compare/contrast
- *Question Cues:* summarize, describe, contrast, predict, associate, distinguish, estimate, differentiate, discuss, report

### Explanation

- knowledgeable and justified account of events, action, and ideas
- see patterns, trends, and relationships between parts
- *Question Cues:* support, confirm, justify, verify, prove, illustrate, use, design, describe, model, predict, show, synthesize, exhibit,

### Interpretation

- making sense of others' work or data using analogy, metaphors, and artistry
- infer meaning and relevance
- *Question cues:* relate, infer, interpret, compose, rewrite, rearrange, evaluate, conclude, make sense of, read between the lines, represent, translate

Adapted from Wiggins, Grant and McTighe, Jay. *Understanding by Design*. Alexandria, VA: Association for Supervision and Curriculum Development, 2006.

## What Does it Mean to Understand? (continued)

### Application

- use information, methods, concepts, theories in new situations and diverse, realistic contexts
- *Question Cues:* apply, demonstrate, calculate, complete, show, solve, change, create, translate, employ, interpret, illustrate, adapt, debug, invent, perform, solve, test

### Perspective

- critical and insightful points of view making assumptions and implications explicit
- create new theories, stories, or applications
- *Question Cues:* analyze, argue, compare, contrast, criticize, infer

### Empathy

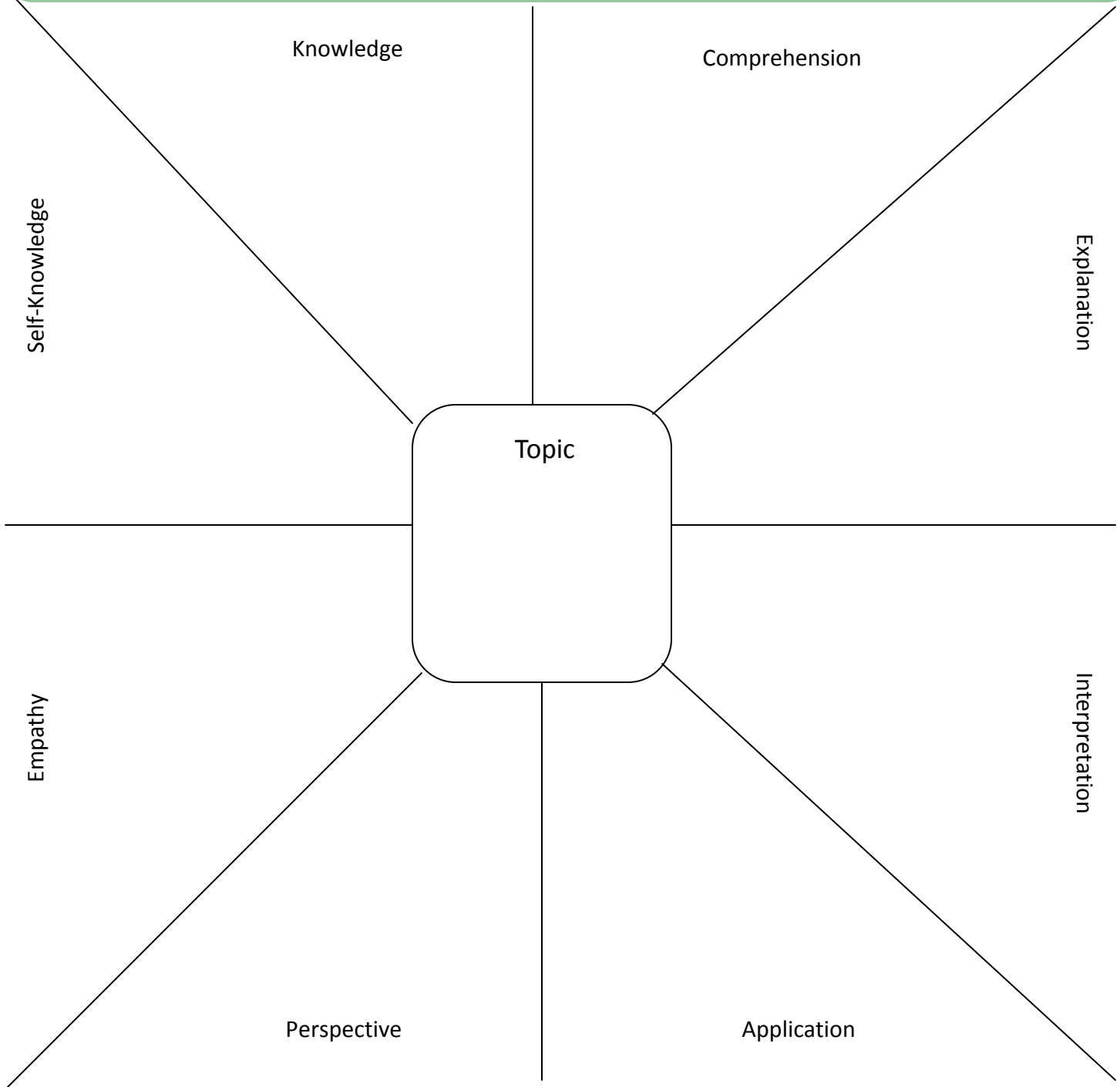
- view a situation from another's point of view or feelings
- find meaning in the experiences or ideas of others
- *Question Cues:* assume the role of, believe, be like, consider, be open to, imagine, relate, role-play

### Self-Knowledge

- self-consciously question our ways of seeing the world beyond ourselves
- look beyond simplistic categories to see unexpected differences, idiosyncrasies, or surprises in people and ideas
- *Question Cues:* be aware of, realize, recognize, reflect, self-assess

Adapted from Wiggins, Grant and McTighe, Jay. *Understanding by Design*. Alexandria, VA: Association for Supervision and Curriculum Development, 2006.

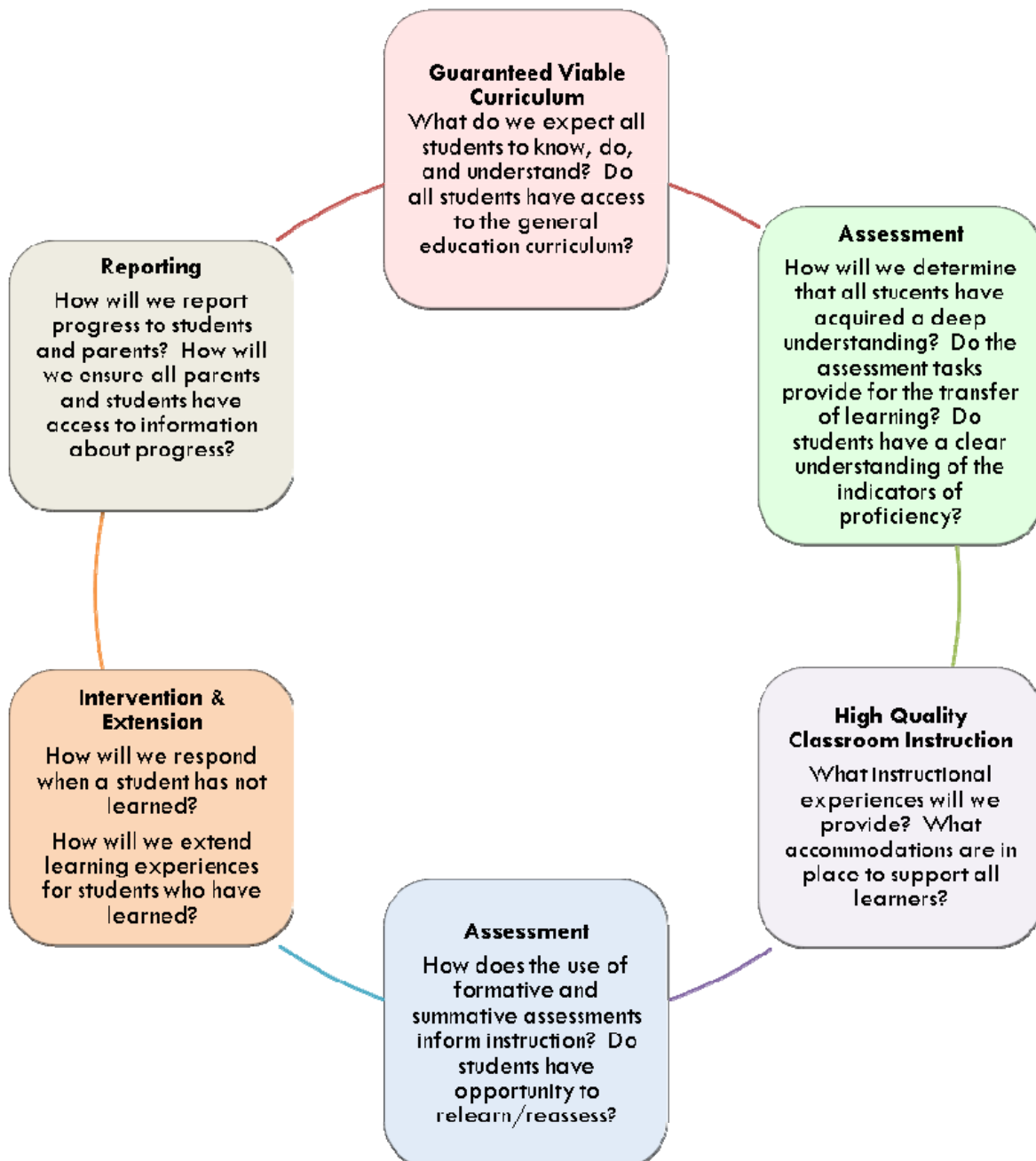
## Levels of Understanding Essential Questions



Adapted from Wiggins, Grant and McTighe, Jay. *Understanding by Design*. Alexandria, VA: Association for Supervision and Curriculum Development, 2006.

## Instructional Framework Making the Connections

A rigorous and challenging standards-based instructional program ensures maximum academic achievement for all students. The Boulder Valley School District Instructional Framework is a graphic representation that demonstrates how all of the components of an instructional program fit together. Teachers should use this framework and its questions to guide instructional planning and decision-making.



## Characteristics of a Boulder Valley School District Standards-based Classroom

### Curriculum

*All Students Have Access to the General Education Curriculum*

- Standards/essential learnings are clearly visible—in writing—in age appropriate student-friendly language
- Continual correlation of curriculum is made to the standards/essential learnings
- Models of high quality products (teacher generated, student generated or both) are provided by the district
- Students and parents are informed of expectations (course syllabus course, standards/essential learnings, grading policy, homework policy, and final culminating activity)
- All students are guaranteed access to the standards/essential learnings
- Lessons and units are developed using a backwards design process
- Suggested timelines are followed

### Instruction

*Quality Instruction Demands Student-Teacher Collaboration in the Learning Process*

Instruction focuses on standards/essential learnings/curriculum

- Clear and high expectation for all students
- Instruction driven by standards/curriculum, not materials or a published program
- Frequent, timely, meaningful feedback of student accomplishment

Instruction supports equity with multiple opportunities to learn through grouping, scaffolding, differentiation, and extension

- Teachers use multiple forms of representation are used (e.g., pictures, words, symbols, diagrams, tables, graphs, word walls)

Students actively engage in learning

- Participate in classroom talk (listening, elaborating, clarifying, expanding)
- Apply rigorous, strategic thinking (application, explanation, perspective, interpretation, perspective, empathy, self-knowledge)

## Characteristics of a Boulder Valley School District Standards-based Classroom

### Assessment

*Assessments are Tightly Aligned to the Standards*

- Students and parents are provided with clear descriptions of proficiency
- Classroom grading practices clearly show how students are progressing toward essential learnings/standards
- Grading is based on attainment of the standards
- Student understanding is assessed through multiple types of formative and summative assessments
- Student assessment results are used to make instructional decisions about what direction to take
- Feedback explicitly guides continuous progress toward mastery of the standard and is provided to students in a timely manner
- Opportunities to relearn, reassess, and extend learning are embedded in every classroom
- Teachers collaborate in the design and analysis of common assessments that are aligned to standards
- Students create authentic products and performances for critical audiences

### Learning Environment

*A Healthy Community of Learners Thrives on Collaborative Processes That Value the Input of All Members*

- Positive respectful relationships are evident within the classroom
- Students monitor and manage the quality of their own learning
- Student enrollment shows gender and racial/ethnic diversity
- Verbal and nonverbal cues indicate student engagement
- Teachers plan so that time is used purposefully and efficiently
- Students use time provided purposefully and efficiently
- Students and teachers negotiate and share decisions that positively impact the learning environment
- Teachers help students make connections between community, nation, world, and self
- Teachers show a connectedness with all students, respectful of student diversity and individual differences
- Students believe they are capable of success, take risks to engage in new experiences, and extend skills and habits of mind

## High School Mathematics Essential Learnings

### Algebra I

- $\pi$  Simplifies expressions involving rational numbers, exponents and absolute value and justifies each step using properties of real numbers and laws of exponents
- $\pi$  Investigates and analyzes functions and function families
- $\pi$  Analyzes and represents linear functions and inequalities to solve problems
- $\pi$  Solves quadratic equations both symbolically and graphically
- $\pi$  Uses the language of mathematics to express ideas precisely through reasoning, representations, and communication

### Geometry I

- $\pi$  Applies congruence and similarity correspondences and properties of the figures to find missing parts of geometric figures and provide logical justification
- $\pi$  Uses algebraic, coordinate, and deductive methods to solve problems involving parallel and perpendicular lines and distance and midpoint formulas
- $\pi$  Investigates and identifies properties of polygons and circles
- $\pi$  Solves practical problems involving right triangles by using the Pythagorean Theorem, properties of special right triangles, and right triangle trigonometry
- $\pi$  Uses formulas to solve practical problems involving perimeter, area, surface area, and volume and use appropriate units of measurement
- $\pi$  Construct and judge the validity of a logical argument consisting of a set of premises and a conclusion

## High School Mathematics Essential Learnings

### Algebra II

- π Performs operations on expressions containing complex numbers, rational exponents and complex fractions
- π Recognizes classes of functions including linear, polynomial, absolute value, step, rational, and exponential from multiple representations such as graphical, tabular, and symbolic and converts between these representations
- π Performs operations on and finds solutions for various types of functions including linear, polynomial, absolute value, and rational
- π Solves linear and nonlinear systems of equations and inequalities symbolically and graphically and performs operations on matrices
- π Uses the language of mathematics to express ideas precisely through reasoning, representations, and communication

### Pre-Calculus

- π Investigates and identifies the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and practical problems
- π Applies trigonometric functions and inverse trigonometric functions to solve practical problems
- π Investigates and identifies the characteristics of the conic sections in order to graph these functions and solve equations and practical problems
- π Investigates and applies the properties of arithmetic and geometric sequences and series to solve practical problems
- π Uses the language of mathematics to express ideas precisely through reasoning, representations, and communication

# *Design Templates*



## Unit Design Template

<b>Desired Results</b>	
<b>BVSD Standard(s)/Essential Learnings</b>	
<b>Unit Enduring Understandings</b>	<b>Unit Essential Questions</b>
<b>Students will know.....</b>	<b>Students will be able to.....</b>
<b>Assessment Evidence</b>	
<b>Performance/Transfer Tasks</b>	<b>Other Evidence</b>
<b>Rubric</b>	<b>Student Self-Assessment and Reflection</b>

## Unit Design Template (continued)

### Learning Plans

**Learning Activities**

**Materials**

**Accommodations**

**Technology Integration**

## Unit Design Template

Essential Learning:

Assessment:

### Teaching for Understanding

	Acquire Knowledge	Make Meaning	Transfer
Essential Questions			
Learning Activities			
Materials			
Accommodations			



## Curriculum Map

Month	Standards/Essential Learnings	Assessment	Knowledge Skills	Learning Activities	Accommodations	Materials

## Curriculum Map

	August	September	October	November	December
<b>Standards/ Essential Learnings</b>					
<b>Assessment</b>					
<b>Knowledge</b>					
<b>Skills</b>					
<b>Learning Activities</b>					
<b>Accommodations</b>					
<b>Materials</b>					

## Curriculum Map

	January	February	March	April	May
<b>Standards/ Essential Learnings</b>					
<b>Assessment</b>					
<b>Knowledge</b>					
<b>Skills</b>					
<b>Learning Activities</b>					
<b>Accommodations</b>					
<b>Materials</b>					

## Curriculum Map

Month

Theme:

Unit Guiding Question(s):

Standards	Assessment	Knowledge and Skills	Learning Activities	Accommodations	Materials
Science					
Math					
Reading					
Writing					
Speaking					
Listening					
Social Studies					
Health					

## Curriculum Map

### Year At A Glance

	Reading	Writing	Math	Science	Social Studies	Health	Speaking/Listening
August							
September							
October							
November							
December							
January							
February							
March							
April							
May							

## Curriculum Map

Unit:

Timing:

<b>Essential Questions</b>					
<b>Standards/Essential Learnings</b>					
Notes	Assessments	Knowledge and Skills	Learning Activities	Accommodations	Materials

## Curriculum Map

Unit:

Timing:

<b>Standards/Essential Learnings</b>	
<b>Enduring Understandings</b>	<b>Assessment</b>
<b>Essential Questions</b>	<b>Knowledge and Skills</b>  <b>Learning Activities</b>  <b>Accommodations</b>  <b>Materials</b>



## Curriculum Glossary of Terms

<b>Anchor</b>	An anchor is a sample of work or performance used to set the specific performance standard for each level of proficiency. Anchors contribute to scoring reliability and support students by providing tangible models of quality work.
<b>Assessment</b>	Assessment refers to the act of determining a value or degree.
<b>Authentic assessment</b>	An authentic assessment is one composed of tasks and activities design to simulate or replicate important, real-world challenges. It asks a student to use knowledge in real-world ways, with genuine purposes, audiences, and situational variables. Authentic assessments are meant to do more than “test;” they should teach students what the “doing” of a subject looks like and what kinds of performance challenges are actually considered most important in a field or profession.
<b>Backward Design</b>	An approach to designing a curriculum or unit that begins with the end in mind and designs toward that end. This term is used by Grant Wiggins and Jay McTighe in <i>Understanding by Design</i> .
<b>Benchmark</b>	Clearly demarcated progress points that serve as concrete indicators for a standard.
<b>Big Idea</b>	In <i>Understanding by Design</i> (Wiggins and McTighe, 2005), the core concepts, principles, theories, and processes that should serve as the focal point of the curriculum, instruction, and assessment. Big ideas are enduring and important and transferable beyond the scope of a particular unit.
<b>Concept</b>	A concept is a mental construct or category represented by a word or phrase. Concepts include both tangible objects (chair, telephone) and abstract ideas (bravery, anarchy).
<b>Content Standard</b>	A content standard answers the question, “What a student should know, do or understand?”
<b>Curriculum</b>	The curriculum represents what should be taught. It is an explicit and comprehensive plan that is based on content and process standards.
<b>Curriculum Implementation</b>	Curriculum implementation is putting the curriculum into place.
<b>Curriculum Mapping</b>	Curriculum mapping and webbing are approaches that require teachers to align the curriculum, standards, and learning activities across grade levels, within a grade level to ensure a continuum of learning that makes sense for all students.
<b>Enduring Understanding</b>	Enduring understandings are specific inferences, based on big ideas that have lasting value beyond the classroom. They are full-sentence statements that describe specifically what students will understand about the topic.

## Curriculum Glossary of Terms (continued)

<b>Essential Learnings</b>	Essential Learnings are the backbone of a guaranteed viable curriculum. Essential Learnings are aligned with standards and articulate the skills, content, and concepts determined to be non-negotiable areas of proficiency attainment by all students so that they are prepared for the next year/level of education. The Essential Learnings are the mandated curriculum of the Boulder Valley School District and form the basis upon which summative assessments are created.
<b>Essential Question</b>	An Essential Question lies at the heart of a subject or a curriculum (as opposed to being either trivial or leading) and promotes inquiry and uncoverage of a subject. Essential questions do not yield a single answer, but produce different plausible responses, about which thoughtful and knowledgeable people may disagree. An essential question can be overarching, grade level specific, or unit specific in scope.
<b>Essential Topics, Skills, Processes, Concepts</b>	The topics, skills, processes, and concepts clarify the Essential Learnings, describe indicators of achievement, and inform the selection of formative and summative assessments.
<b>Formative assessment</b>	An assessment is considered formative when the feedback from learning activities is actually used to adapt the teaching to meet the learner's needs.
<b>Guaranteed Viable Curriculum</b>	In researching what works in schools, Robert Marzano (2003), found five school-level factors that promote student achievement. Using the process of statistical effect size analysis, Marzano concluded that a guaranteed and viable curriculum is the most powerful school-level factor in determining overall student achievement. Marzano defines a guaranteed and viable curriculum as a combination of opportunity to learn (guaranteed) and time to learn (viable). According to Marzano, students have the opportunity to learn when they study a curriculum that clearly articulates required standards to be addressed at specific grade levels and in specific courses. A curriculum is viable when the number of required standards is manageable for a student to learn to a level of mastery in the time provided (usually a semester, trimester, or year).
<b>Learning Activities</b>	These represent the experiences and instruction that will enable students to achieve the desired results such as materials, projects, lectures, videos, homework, assignments, presentations, accommodations, and vocabulary.
<b>Performance Task</b>	A performance task uses one's knowledge to effectively act or bring to fruition a complex product that reveals one's knowledge and expertise.
<b>Prerequisite knowledge and skill</b>	The knowledge and skill required to successfully perform a culminating tasks or achieve an understanding. These typically identify discrete knowledge and know-how required to put everything together in a meaningful, final performance.

## Curriculum Glossary of Terms (continued)

<b>Processes</b>	Processes include all the strategies, decisions, and sub-skills a student uses in meeting the content standard.
<b>Product</b>	The tangible and stable result of a performance and the processes that led to it. The product is valid for assessing the student's knowledge to the extent that success or failure in producing the product reflects the knowledge taught and being assessed.
<b>Rubric</b>	A scoring tool that rates performance according to clearly stated levels of criteria and enables students to self-assess. A rubric answers the question, <i>What does understanding or proficiency for an identified result look like?</i> The scales can be numeric or descriptive.
<b>Scope and Sequence</b>	Scope refers to the breadth and depth of content to be covered in a curriculum at any one time (e.g. week, term, year, over a student's school life). Sequence refers to the order in which content is presented to learners over time. The order in which you do it. Together a scope and sequence of learning bring order to the delivery of content, supporting the maximizing of student learning and offering sustained opportunities for learning. Without a considered scope and sequence there is the risk of ad hoc content delivery and the missing of significant learning.
<b>Strategies</b>	Strategies are procedures, methods, or techniques to accomplish an essential learning.
<b>Summative assessment</b>	An assessment is considered summative when the feedback is used as a summary of the learning up to a given point in time.



# Pre-Calculus Curriculum Essentials





## Boulder Valley School District Mathematics Background

The National Mathematics Advisory Panel's Final Report (2008) and the National Council of Teachers of Mathematics' *Curriculum Focal Points* (2006) provided the structure and guiding principals for the BVSD revision of the mathematics curriculum. The National Mathematics Advisory Panel's Report delineated the prerequisite knowledge necessary for success in algebra and key algebra topics students should learn. The *Curriculum Focal Points* recommended focusing each year on relatively few but highly important topics a change from the mile wide inch deep curriculums in use across the country. As a result of these recommendations, this BVSD mathematics curriculum asks students to

focus on only a few essential learnings designed to prepare students for learning in future mathematical courses and careers. By focusing on only a few essential learnings students receive extended experiences with key ideas that build deep understanding, fluency with skills, and the ability to generalize and transfer knowledge to future learning.

## Boulder Valley School District Mathematics Technology & Information Literacy

The National Council of Teachers of Mathematics Principles and Standards (2000) states that "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24). The use of technology in BVSD mathematics classrooms is chosen carefully and integrated consistently throughout the curriculum in order to enhance learning and support effective teaching. Technology is not a replacement for understanding of key ideas and skills, however it can and should be used to expand the topics that are accessible to all students at each grade level. The use of technology

provides opportunities for students to focus on mathematical concepts, create conjectures, generalize their thinking, and create justifications. Technology has the potential for extending the boundaries of the classroom and providing students with opportunities for increased practice and access to novel problems deemed inaccessible prior to technology.

## Boulder Valley School District Mathematics Process Standards

The Process Standards of the National Council of Teachers of Mathematics are a key component to the BVSD mathematics curriculum. They indicate the ways in which students should acquire and use their content knowledge. The five process standards in mathematics are problem solving, reasoning communication, connections, and representations. These processes are an integral part of all mathematics learning and teaching. A mathematical learning experience focused on the five process standards prepares students with the processes necessary for continued learning in future mathematical courses and careers. Therefore every process standards should be an integral part of the learning and assessment of every essential learning.

### Communication

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely

### Representations

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

### Reasoning

- Recognize reasoning and proof and fundamental aspects of math
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments
- Select and use various types of reasoning and methods of proof

### Connections

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

### Problem Solving

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

## Boulder Valley School District Mathematics Content Standards

### Mathematics Standard 1 (Number)

*Students develop number sense and use number relationships to solve problems. They communicate their reasoning used to solve these problems.*

### Mathematics Standard 2 (Algebra)

*Students use algebraic methods to solve problems by exploring, modeling, and describing patterns and relations involving numbers, shapes, data, and graphs. They communicate their reasoning used to solve these problems.*

### Mathematics Standard 3 (Data Analysis and Probability)

*Students use data collection and analysis, statistics, and probability to solve problems. They communicate their reasoning used to solve these problems and accurately display the data in a way that conclusions can be drawn.*

### Mathematics Standard 5 (Measurement)

*Students use a variety of measurement tools, techniques, and systems to solve problems. They communicate their reasoning used to solve these problems.*



### Mathematics Standard 4 (Geometry)

*Students use geometric concepts, properties, and relationships in one, two, and three dimensions to model and solve problems. They communicate their reasoning used to solve these problems.*

### Mathematics Standard 6 (Operations)

*Students make connections between concepts and procedures to effectively use computational skills to solve problems. They use appropriate techniques for the problem or situation (for example: estimation, mental math, paper and pencil, calculators, computers). They communicate their reasoning used to solve these problems.*

## Mathematics Overarching Enduring Understandings and Essential Questions

### Overarching Enduring Understanding

- Mathematics can be used to solve problems outside of the mathematics classroom.
- Mathematics is built on reason and always makes sense.
- Reasoning allows us to make conjectures and to prove conjectures.
- Classifying helps us to build networks of mathematical ideas.
- Precise language helps us express mathematical ideas and receive them.

### Overarching Essential Question

- Is your plan working?
- Do you need to reconsider what you are doing?
- How are solving and proving different?
- How are showing and explaining different?
- How do you know when you have proven something?
- What does it take to verify a conjecture?
- How do you develop a convincing argument?
- Why do we classify?
- Why do we classify numbers, geometric objects and functions?
- How do you make sense of different strategies? How do you determine their strengths and weaknesses?
- How do you determine similarities and differences?

## Boulder Valley School District Content Standards and Pre-Calculus Essential Learnings

**Standard 1 (Number):** *Students develop number sense and use number relationships to solve problems. They communicate their reasoning used to solve these problems.*

**Standard 6 (Operations):** *Students make connections between concepts and procedures to effectively use computational skills to solve problems. They use appropriate techniques for the problem or situation (for example: estimation, mental math, paper and pencil, calculators, computers). They communicate their reasoning used to solve these problems.*

**No essential learning at this grade level. Addressed through connections.**

**Standard 3 (Data Analysis and Probability):** *Students use data collection and analysis, statistics, and probability to solve problems. They communicate their reasoning used to solve these problems and accurately display the data in a way that conclusions can be drawn.*

**No essential learning at this grade level. Addressed through connections.**

**Standard 2 (Algebra):** *Students use algebraic methods to solve problems by exploring, modeling, and describing patterns and relations involving numbers, shapes, data, and graphs. They communicate their reasoning used to solve these problems.*

**To meet this standard, a Pre-Calculus student:**

- √ Investigates and identifies the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and practical problems
- √ Applies trigonometric functions and inverse trigonometric functions to solve practical problems
- √ Investigates and identifies the characteristics of the conic sections in order to graph these functions and solve equations and practical problems.
- √ Investigates and applies the properties of arithmetic and geometric sequences and series to solve practical problems.
- √ Investigates the characteristics of functions including the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity.

**Standard 4 (Geometry):** *Students use geometric concepts, properties, and relationships in one, two, and three dimensions to model and solve problems. They communicate their reasoning used to solve these problems.*

**Standard 5 (Measurement):** *Students use a variety of measurement tools, techniques, and systems to solve problems. They communicate their reasoning used to solve these problems.*

**No essential learning at this grade level. Addressed through connections.**

**Mathematical Processes:** *Students use the mathematical processes of problem solving, reasoning and proof, communication, connections and representations to acquire and use mathematical knowledge.*

**To meet this process, a Pre-Calculus student:**

- √ Uses the language of mathematics to express ideas precisely through reasoning, representations, and communication.

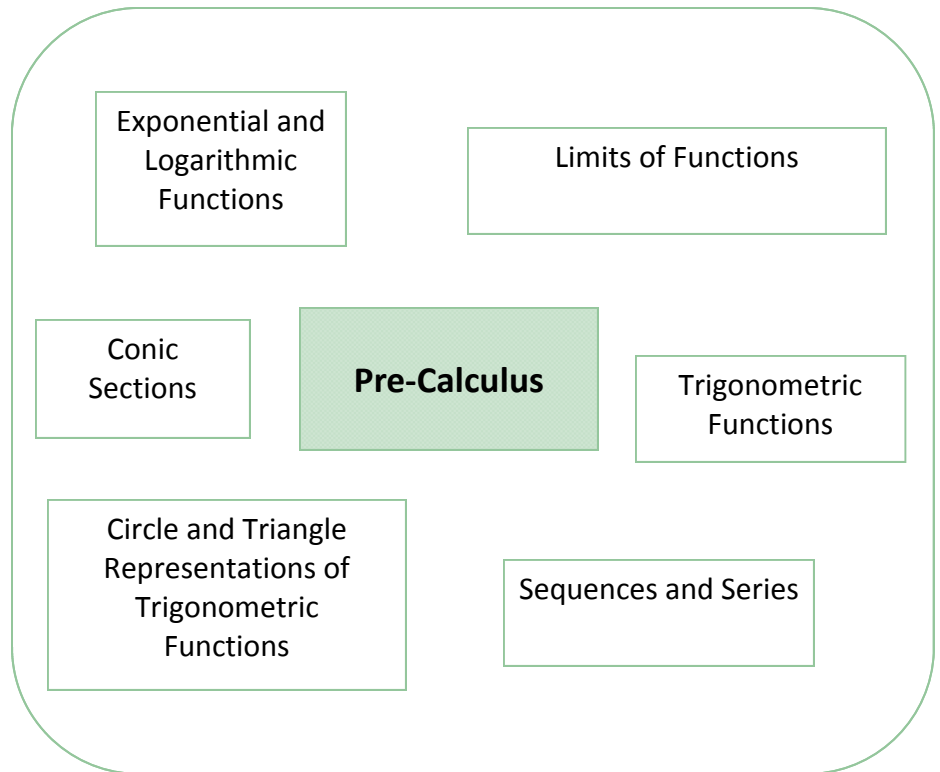
## BVSD Pre-Calculus Overview

### Course Description

Mathematics at the Pre-Calculus level focuses on exponential/logarithmic functions, trigonometric functions, conic sections, sequences and limits of functions. Problem solving, representations, reasoning, communication, and connections within and outside of mathematics underline all of the teaching and learning at the Pre-Calculus level.

### Effective Components of a Pre-Calculus Program

- Structures for mental math are utilized regularly
- Mathematical ideas are connected to experiences and prior knowledge
- Misconceptions are addressed quickly
- Students communicate using mathematical language
- Varied representations and models are used to learn a concept
- Strategies and conjectures are justified and explained



### Assessment

- √ Teacher Created Assessments
- √ Math Notebooks

### Essential Questions

- How can change be described mathematically?
- How can transformations be described mathematically?

### Technology Integration & Information Literacy

- ① Employs technology to visualize, model, investigate, and extend mathematical reasoning
  - ① Uses technology to practice and assess needed math skills
  - ① Displays, presents, creates and/or shares learning in mathematics using available technology
  - ① Accesses school library, teacher-librarian, Internet, and other age-appropriate mathematical resources
  - ① Uses technology responsibly
- For information about available core software, relevant web resources, and other integration activities, please use the following website: <http://bvsd.org/iteach/integration>

## Essential Learnings

### Essential Knowledge, Skills, Topics, Processes, and Concepts

#### Mathematics Standard 2 (Algebra)

*Students use algebraic methods to solve problems by exploring, modeling, and describing patterns and relations involving numbers, shapes, data, and graphs. They communicate their reasoning used to solve these problems.*

#### Enduring Understanding

Change is fundamental to understanding functions.

#### Essential Question

How can change be described mathematically?

### Essential Learnings

Essential Knowledge, Skills, Topics, Processes, and Concepts	<b>PCM1</b>	<b>Investigates and identifies the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and practical problems</b>
	a	Identifies exponential and logarithmic functions from an equation and graph and identifies asymptotes, intercepts, domain and range
	b	Identifies common and natural logarithms and define e, and know its approximate value
	c	Writes logarithmic equations in exponential form and vice versa
	d	Uses laws of exponents and logarithms to solve equations and simplify expressions
	e	Models practical problems using exponential and logarithmic functions

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## Essential Learnings

### Essential Knowledge, Skills, Topics, Processes, and Concepts

#### Mathematics Standard 2 (Algebra) (continued)

*Students use algebraic methods to solve problems by exploring, modeling, and describing patterns and relations involving numbers, shapes, data, and graphs. They communicate their reasoning used to solve these problems.*

#### Enduring Understanding (continued)

Change is fundamental to understanding functions.

#### Essential Question (continued)

How can change be described mathematically?

### Essential Learnings

Essential Knowledge, Skills, Topics, Processes, and Concepts	<b>PCM2</b>	<b>Applies trigonometric functions and inverse trigonometric functions to solve practical problems</b>
	a	Defines the six triangular and circular trigonometric functions of an angle in standard position and makes connections between the triangular and circular trigonometric functions
	b	Recognizes the connections between coordinates of points on a unit circle and coordinate geometry; cosine and sine values; and lengths of special right triangles
	c	Finds the trigonometric function values of any angle in degrees or radians and use inverse trigonometric functions to find angle measures when trigonometric values are given
	d	Finds the domain and range of inverse trigonometric functions and uses restrictions on the domains of the inverse trigonometric functions to find values of the inverse functions
	e	Sketches the graph of a trigonometric function in standard form {e.g., $y = A \sin(Bx + C) + D$ } and describes the effects of changing $A$ , $B$ , $C$ or $D$ of the equation
	f	Determines the amplitude, period, phase shift, and vertical shift of trigonometric functions from the equation of the function and the graph of the function
	g	Uses trigonometric identities to make algebraic substitutions to simplify and verify trigonometric identities including reciprocal identities, Pythagorean identities, sum and difference identities, double angle identities, and half-angle identities
	h	Uses trigonometric functions, Pythagorean Theorem, Law of Sines and Law of Cosines to solve practical problems

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## Essential Learnings

### Essential Knowledge, Skills, Topics, Processes, and Concepts

#### Mathematics Standard 2 (Algebra) (continued)

*Students use algebraic methods to solve problems by exploring, modeling, and describing patterns and relations involving numbers, shapes, data, and graphs. They communicate their reasoning used to solve these problems.*

#### Enduring Understanding

An objects location on a plane or in space can be described quantitatively

#### Essential Question

How can transformations be described mathematically?

### Essential Learnings

Essential Knowledge, Skills, Topics, Processes, and Concepts	<b>PCM3</b>	<b>Investigates and identifies the characteristics of the conic sections in order to graph these functions and solve equations and practical problems</b>
	a	Identifies equations of and be able to graph conic sections in Cartesian and Polar form (e.g., circles, ellipses, parabolas, and hyperbolas)
	b	Identifies foci and vertices of ellipses and hyperbolas
	c	Graphs conic sections from standard equations and recognizes the need for a hyperbola to approach the asymptotes
	d	Uses completing the square to put the equations of conic sections in standard form
	e	Constructs circles and ellipses using informal methods and the definition of circles and ellipses (i.e., string and thumbtacks)
	f	Models practical problems involving the conic sections (e.g., orbits of planets)

## Essential Learnings

### Essential Knowledge, Skills, Topics, Processes, and Concepts

#### Mathematics Standard 2 (Algebra) (continued)

*Students use algebraic methods to solve problems by exploring, modeling, and describing patterns and relations involving numbers, shapes, data, and graphs. They communicate their reasoning used to solve these problems.*

#### Enduring Understanding

Numbers or objects that repeat in predictable ways can be described or generalized.

#### Essential Question

How are patterns of change related to the behavior of functions?

### Essential Learning

Essential Knowledge, Skills, Topics, Processes, and Concepts	<b>PCM4</b>	<b>Investigates and applies the properties of arithmetic and geometric sequences and series to solve practical problems</b>
	a	Distinguish between a sequence and a series
	b	Recognize and describe patterns in a sequence and distinguish between arithmetic and geometric sequences
	c	Use and interpret the notations $\Sigma$ , $n$ , $n^{\text{th}}$ term, and $a_n$ .
	d	Write the first $n$ terms in an arithmetic or geometric sequence
	e	Determines the recursive pattern of a sequence
	f	Given the formula, find $a_n$ (the $n^{\text{th}}$ term) for an arithmetic or geometric sequence
	g	Given formulas, find the sum, $S_n$ , of the first $n$ terms of an arithmetic or geometric series, including infinite series

## Essential Learnings

### Essential Knowledge, Skills, Topics, Processes, and Concepts

#### Mathematics Standard 2 (Algebra) (continued)

*Students use algebraic methods to solve problems by exploring, modeling, and describing patterns and relations involving numbers, shapes, data, and graphs. They communicate their reasoning used to solve these problems.*

#### Enduring Understanding

Change is fundamental to understanding functions.

#### Essential Question

How can change be described mathematically?

### Essential Learnings

Essential Knowledge, Skills, Topics, Processes, and Concepts	<b>PCM5</b>	<b>Investigates the characteristics of functions including the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity</b>
	a	Recognizes polynomial and rational functions from equations and graphs by identifying the zeros, upper and lower bounds, y-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, points of discontinuity, end behavior, and maximum and minimum points
	b	Finds the composition of functions and the domain and range of composite functions
	c	Finds the inverse of a function graphically and symbolically and determines its domain and range
	d	Describes the continuity of a function and investigates the continuity of absolute value, step, rational and piece-wise-defined functions
	e	Defines the limit of a function as the value approached by $f(x)$ as $x$ approaches a given value or infinity with respect to rational functions
	f	Finds the limit of a function algebraically and numerically and verifies graphically

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## Essential Learnings

### Essential Knowledge, Skills, Topics, Processes, and Concepts

#### Mathematical Processes

*Students use the mathematical processes of problem solving, reasoning and proof, communication, connections and representations to acquire and use mathematical knowledge.*

#### Enduring Understanding

Mathematics can be used to solve problems outside of the mathematics classroom.  
 Mathematics is built on reason and always makes sense.  
 Reasoning allows us to make conjectures and to prove conjectures.  
 Precise language helps us express mathematical ideas and receive them.

#### Essential Question

Is your plan working? Do you need to reconsider what you are doing?  
 How are showing and explaining different?  
 How do you develop a convincing argument?  
 How do you make sense of different strategies? How do you determine their strengths and weaknesses?

## Essential Learnings

Essential Knowledge, Skills, Topics, Processes, and Concepts	<b>PCM5</b>	<b>Uses the language of mathematics to express ideas precisely through reasoning, representations, and communication</b>
		a Selects, applies, and translates among mathematical representations to solve problems and justifies the reasonableness of solutions
		b Creates and uses representations to organize, record and explain mathematical ideas clearly to peers, teachers and others
		c Analyzes and evaluates the mathematical thinking, strategies and arguments of peers, teachers and others
		d Recognizes, uses, and explains connections among mathematical ideas in contexts both inside and outside of mathematics classrooms
		e Develops, tests and explains mathematical conjectures using inductive and deductive reasoning and proves formulas using mathematical induction
		f Recognizes and utilizes key academic vocabulary relevant to mathematics in verbal and written communication
		g Given a real-world problems selects an appropriate method to solve the problem by determining if the information provided is sufficient, insufficient or extraneous
		h Creates and illustrates a real-world problem from a given math sentence

## Suggested Timelines

Topic	Suggested Timeframe
Investigates and identifies the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and practical problems.	Six weeks of instruction over the course of the school year.
Applies trigonometric functions and inverse trigonometric functions to solve practical problems.	Six weeks of instruction over the course of the school year.
Investigates and identifies the characteristics of the conic sections in order to graph these functions and solve equations and practical problems.	Seven weeks of instruction over the course of the school year.
Investigates the characteristics of functions including the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity.	Seven weeks of instruction over the course of the school year.
Investigates and applies the properties of arithmetic and geometric sequences and series to solve practical problems.	Four weeks of instruction over the course of the school year.



## Mathematics Scope & Sequence K-2

Standard	K	1	2
<b>Number and Operation</b>	Numbers to 20 Combinations to 5 Coin Identification	Numbers to 100 Ordinal Numbers Combinations to 10 Skip Counts Addition and Subtraction Contexts Values of Coins	Numbers to 1000 Addition and Subtraction Facts Estimation of Sums and Differences Halves, Thirds and Fourths Combinations of Coins
<b>Algebra</b>	Repeating patterns	Repeating Patterns Number Equality Odd and Even Commutative Property of Addition Inverse Relationship of Addition and Subtraction	Growing and Repeating Patterns Commutative Property of Addition Qualitative and Quantitative Change
<b>Geometry and Measurement</b>	Basic Shapes Days of Week Relative Location Non-Standard Measurement	Time to half hour Months of the Year Inch and Foot Shapes and Solids Tessellations	Time to five minute Inches and Centimeters Attributes of Shapes Symmetry Rectangular Arrays
<b>Data Analysis and Probability</b>	Data Collection	Data Collection Data Interpretation	Data Analysis Questions Data Collection Graphical Representations Data Interpretation Mode Simple Probability
<b>Mathematical Processes</b>	Communication Representations Reasoning and Proof Problem Solving Connections	Communication Representations Reasoning and Proof Problem Solving Connections	Communication Representations Reasoning and Proof Problem Solving Connections

## Mathematics Scope & Sequence 3-5

Standard	3	4	5
<b>Number and Operation</b>	Numbers to 10,000 Multi-Digit Addition and Subtraction Multiplication and Division Context Fraction Models Money Addition and Subtraction	Number to 100,000 Multiplication Facts Multi-Digit Multiplication Multiples and Factors Prime and Composite Divisibility Rules Rational Number Comparison	Multi-Digit Multiplication and Division Remainders as Decimals and Fractions Addition and Subtraction of Rational Numbers
<b>Algebra</b>	Growing Patterns Properties of Addition Odd and Even In/Out Tables Inverse Relationship of Multiplication and Division	Properties of Multiplication In/Out Tables Tabular and Graphical Patterns Distributive Property	Growth Pattern Equations Rates of Change Variables Rational Number Conjectures
<b>Geometry and Measurement</b>	Measurement Time to minute Perimeter Parallel Congruence Points, Lines and Rays	Area and Perimeter Distance and Scale Coordinate Graphing Measurement to half unit	Angles Transformations Measurement to quarter unit 2-D Representations of Solids
<b>Data Analysis and Probability</b>	Combination Problems Graphical Representations Data Interpretation	Fairness Sample Space Likelihood of Events Data Analysis Mode, Median, Range Clusters, Outliers	Categorical and Numerical Questions Data Collection Methods Graphical Representations Data Descriptions and Analysis
<b>Mathematical Processes</b>	Communication Representations Reasoning and Proof Problem Solving Connections	Communication Representations Reasoning and Proof Problem Solving Connections	Communication Representations Reasoning and Proof Problem Solving Connections

## Mathematics Scope & Sequence 6-8

Standard	6	7	8
<b>Number and Operation</b>	Positive Rational Numbers Addition, Subtraction Multiplication, Division Estimation Percent Problems Properties of Positive Rational Numbers	Integers Negative Rational Numbers Addition, Subtraction Multiplication, Division Properties of Rational Numbers Order of operations Distributive Property Ratios and Proportion	Exponents Scientific Notation Irrational Numbers Estimation
<b>Algebra</b>	Represent and Analyze Patterns Variables and Expressions Descriptions of Change	Linear Relationships Analysis of Change Simple Linear Equation Solutions	Slope Y-intercepts Linear Functions Functional Relationships
<b>Geometry and Measurement</b>	Area and Perimeter Formulas Estimation of Irregular Areas Maximum/Minimum Area and Perimeter Problems Circumference and Area of Circles	Similarity Similar Figures Scale Factor Linear Dimensions, Angles, Perimeters and Areas Four Quadrant Coordinate Graphs	Pythagorean Theorem Rectangular Prisms Pyramids Volume Surface Area
<b>Data Analysis and Probability</b>	Measures of Central Tendency Mean, Median, Mode Graphical Representations Data Analysis Hypotheses and Conclusions	Coordinate Graphing Data Analysis Simple Probability	Population and Samples Sample Size Random Samples Probability Sample Distributions
<b>Mathematical Processes</b>	Communication Representations Reasoning and Proof Problem Solving Connections	Communication Representations Reasoning and Proof Problem Solving Connections	Communication Representations Reasoning and Proof Problem Solving Connections

## Mathematics Scope & Sequence 9-12

Standard	Algebra	Geometry	Algebra 2	Pre-Calculus
<b>Number and Operation</b>	Real Number Operations Order of Operations Law of Exponents Absolute Value Scientific Notation		Complex Number Operations	
<b>Algebra</b>	Polynomial Operations Functions Linear Equations/ Inequalities Quadratic Equations Systems of Equations		Families of Functions Polynomial, Radical Absolute Value, Rational, Systems of Linear and Nonlinear Equations Matrices Solutions of Linear, Polynomial, Rational and Radical	Exponential and Logarithmic Functions Limits of Functions Conic Sections Sequences and Series Circle and Triangle Representations of Trigonometric Functions
<b>Geometry and Measurement</b>		Congruence/ Similarity Properties of Polygons Properties of Circles Pythagorean Theorem Trigonometric Ratios Perimeter, Area, Volume Unit Conversions Proof and Argument		
<b>Data Analysis and Probability</b>				
<b>Mathematical Processes</b>	Communication Representations Reasoning and Proof Problem Solving Connections	Communication Representations Reasoning and Proof Problem Solving Connections	Communication Representations Reasoning and Proof Problem Solving Connections	

## Mathematics Glossary of Terms

Absolute value	A number's distance from zero on a number line. The absolute value of $-6$ , shown as $ 6 $ , is 6, and the absolute value of 6, shown as $ 6 $ , is 6.
Algebraic methods	The use of symbols to represent numbers and signs to represent their relationships
Algorithm	A step-by-step procedure
Box plot (also called a box-and-whiskers plot)	A graphic method for showing a summary of data using median, quartiles, and extremes of data. A box plot makes it easy to see where the data are spread out and where they are concentrated. The longer the box, the more the data are spread out.
Capacity	The volume of a container given in units of liquid measure. The standard units of capacity are the liter and the gallon.
Combinations	Subsets chosen from a larger set of objects in which the order of the items doesn't matter (for example, the number of different committees of three that can be chosen from a group of twelve members).
Complex numbers	Numbers that can be written in the form $a + bi$ , for example, $-2.7 + 8.9i$ , where $a$ and $b$ are real numbers and $i = \sqrt{-1}$
Congruent or the concept of congruence	Two figures are said to be congruent if they are the same size and shape.
Coordinate geometry	Geometry based on the coordinate system
Coordinate system (also called rectangular coordinate system)	A method of locating points in the plane or in space by means of numbers. A point in a place can be located by its distances from both a horizontal and a vertical line called the axes. The horizontal line is called the x-axis. The vertical line is called the y-axis. The pairs of number are called ordered pairs. The first number, called the x-coordinate, designates the distance along the horizontal axis. The second number, called the y-coordinate, designates the distance along the vertical axis. The point at which the two axes intersect has the coordinates $(0,0)$ and is called the origin.
Conjecture	A statement that is to be shown true or false. A conjecture is usually developed by examining several specific situations.

## Mathematics Glossary of Terms (continued)

Dilation	A transformation that either enlarges or reduces a geometric figure proportionally.
Exponential function	A function that has an equation of the form $y=a^x$ . These functions are used to study population growth or decline, radioactive decay, and compound interest.
Exponent	A number used to tell how many times a number or variable is used as a factor. For example, $5^3$ indicates that 5 is a factor 3 times, that is, $5 \times 5 \times 5$ . The value of $5^3$ is 125.
Function	A function consists of a domain and a rule. The domain is a set of real numbers. The rule assigns to each number in the domain one and only one number
Integers	The set of numbers consisting of the counting numbers (that is, 1, 2, 3, 4, 5, ...), their opposites (that is, negative numbers, -1, -2, -3, ...), and zero.
Irrational numbers	The set of numbers which cannot be represented as fractions. Examples are $\sqrt{2}$ , the cube root of 29, e, and $1/4$ .
Linear function	A function that has a constant rate of change.
Logarithm	Alternate way to express an exponent. For example, $\log_2 8=3$ is equivalent to 2 to the third power equals 8.
Matrix (pl. matrices)	A rectangular array of numbers (or letters) arranged in rows and columns
Measures of central tendency	Numbers which in some sense communicate the “center” or “middle” of a set of data. The mean, median, and mode of statistical data are all measures of central tendency.
Measures of variability	Numbers which describe how spread out a set of data is, for example, range and quartile.
Mental arithmetic	Performing computations in one’s head without writing anything down. Mental arithmetic strategies include finding pairs that add up to 10 or 100, doubling, and halving.
Model	To make or construct a physical or mathematical representation.
Number sense	An understanding of number. This would include number meanings, number relationships, number size, and the relative effect of operations on numbers.

## Mathematics Glossary of Terms (continued)

Open sentence	A statement that contains at least one unknown. It becomes true or false when a quantity is substituted for the unknown. For example, $3 + x = 5$
Optimization problems	Real-world problems in which, given a number of constraints, the best solution is determined. For example, finding the best number of nonstop flights from Denver to San Francisco given the cost of fuel, number of passengers, number of crew required, etc.
Patterns	Regularities in situations such as those in nature, events, shapes, designs, and sets of number (for example, spirals, on pineapples, geometric designs in quilts, the number sequence 3, 6, 9, 12, ...)
Permutations	All possible arrangements of a given number of items in which the order of the items makes a difference. For example, the different ways that a set of four books can be placed on a shelf.
Prime number	A counting number that can only be evenly divided by two different numbers, 1 and the number itself. The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.
Probability	The likeliness or chance of an event occurring.
Problem-solving situations	Contexts in which problems are presented that apply mathematics to practical situations in the real world, or problems that arise from the investigation of mathematical ideas.
Quadratic function	A function that has an equation of the form $y = ax^2 + bx + c$ , where $a \neq 0$ . These functions are used to describe the flight of a ball and the stream of water from a fountain.
Rational numbers	A number that can be expressed in the form $a/b$ , where $a$ and $b$ are integers and $b \neq 0$ , for example, $3/4$ , $2/1$ , or $11/3$ . Every integer is a rational number, since it can be expressed in the form $a/b$ , for example, $5 = 5/1$ . Rational numbers may be expressed as fractional or decimal numbers, for example, $3/4$ or $.75$ . Finite decimals, repeating decimals, and mixed numbers all represent rational numbers.
Real numbers	All rational and irrational numbers
Real-world problems (also called real-world experiences)	Quantitative problems that arise from a wide variety of human experience which may take into consideration contributions from various cultures (for example, Mayan or American pioneers), problems from abstract mathematics, and applications to various careers (for example, making change or calculating the sale price of an item).

## Mathematics Glossary of Terms (continued)

Reflection (also called a flip)	A transformation which produces the mirror image of a geometric figure.
Rotation (also called a turn)	A transformation which turns a figure about a point a given number of degrees.
Scatter plots (also called scatter diagram or scattergram)	A graph of the points representing a collection of data.
Scientific notation	A short-hand way of writing very large or very small numbers. A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10, for example, $4.53 \times 10^3 = 4350$ .
Similarity	Objects or figures that are the same shape are similar figures. They are not necessarily the same size. If two figures are similar, we say that there is similarity between the figures.
Spatial visualization (also called spatial reasoning)	A type of reasoning in which a person can draw upon one's understanding of relationships in space, the three-dimensional world. For example, spatial reasoning is demonstrated by one's ability to build a three-dimensional model of a building shown in a picture. A person who uses spatial visualization is said to have spatial sense.
Square root	That number which when multiplied by itself produces the given number, for example, 5 is the square root of 25, because $5 \times 5 = 25$ .
Stem-and-leaf plot	A frequency distribution made by arranging data. It is one way of visually portraying data that is frequently used in newspapers and magazines because it provides an efficient way of showing information as well as comparing different sets of data.
Symmetry	The correspondence in size, form, and arrangement of parts on opposite sides of a plane, line, or point. For example, a figure that has line symmetry has two halves which coincide if folding along its line of symmetry.
Transformation	The process of changing one configuration or expression into another in accordance with a rule. Common geometric transformations include translations, rotation, and reflections.
Translation (also called a slide)	A transformation that moves a geometric figure by sliding. Each of the points of the geometric figure moves the same distance in the same direction.
Trigonometric ratios	The ratios of the lengths of pairs of sides in a right triangle. There are three basic trigonometric ratios used in trigonometry: sine (sin), cosine (cos), and tangent (tan).

## Mathematics Glossary of Terms (continued)

Variable	A quantity that may assume any one of a set of values. In the equations $2x + y = 9$ , $x$ and $y$ are variables.
Vector	A quantity which has both magnitude and direction. Vectors may be interpreted as physical quantities such as velocity and force.
Volume	The measure of the interior of a three-dimensional figure. A unit for measuring volume is the cubic unit.